B.SC. THIRD SEMESTER (PROGRAMME) EXAMINATION, 2021

Subject: Mathematics **Course ID: 32110**

Course Code: SP/MTH/304/SEC-1 **Course Title: Logic and Sets**

Time: 2 Hours **Full Marks: 40**

<u>The figures in the margin indicate full marks.</u>

Notations and symbols have their usual meaning

1. Answer any five questions: $2 \times 5 = 10$

(a) A relation ρ is defined on the set $\mathbb{Z}$ by "$x\rho y$ if and only if $x + y$ is odd" for $x, y \in \mathbb{Z}$. Examine if ρ is an equivalence relation on $\mathbb{Z}$.

(b) Find all equivalence relations on the set $A = \{1, 2, 3\}$.

(c) If $A = \{1, 2\}, B = \{2,3\}$ and $C = \{3,4\}$, then find $A \times (B \cup C)$.

(d) If A and B are two non-empty sets, then prove that $A \cup B = (A - B) \cup B$.

(e) If X, Y, A are three non-empty sets such that $A \cap X = A \cap Y$ and $A \cup X = A \cup Y$, then prove that $X = Y$.

(f) If $n(A)$ and $n(B)$ denote the number of elements in the finite sets A and B respectively and $n(A) = p$ and $n(B) = q$, then find number of relations from A to B?

(g) Construct a truth table for the statement formula $(\sim a \wedge \sim b)$.

(h) Find the negation of the following statement: $\exists x\ p(x) \wedge y\ q(y)$.

2. Answer any four questions: $5 \times 4 = 20$

(a) (i) Prove that $A - (B \cap C) = (A - B) \cup (A - C)$, where A, B, C are subsets of a set X.

(ii) Show that $A \cap B = A$ if and only if $A \subseteq B$. 3+2

(b) (i) If $n(A)$ and $n(B)$ denote the number of elements in the finite sets A and B respectively, then prove by Venn diagram, that

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

(ii) If $A \cup B = B$, then prove that $A = \phi$ or $A \subset B$ 3+2

(c) A relation ρ is defined on $\mathbb{Z}$ by "$x\rho y$ iff $2x + 3y$ is divisible by 5" then prove that ρ is an equivalence relation on $\mathbb{Z}$.

(d) (i) Let ρ be an equivalence relation on a set X and $a, b \in X$. Then prove that the classes $cl(a)$ and $cl(b)$ are either equal or disjoint.

(ii) The relation ρ is defined on the set $\mathbb{R}$ by "$x\rho y$ iff x is less than or equal to y" for all $x, y \in \mathbb{R}$. Examine if ρ is a partial order relation. 3+2

(e) It is known that in a university 60% of professors play tennis, 50% of them play bridge,70% jog, 20% play tennis and bridge, 40% play bridge and jog and 30% play tennis and jog. If someone claimed that 20% professors' jog and play tennis and bridge, would you believe his claim? Why? 5

(f) (i) Construct a truth table for the statement form:

$(p \vee q) \leftrightarrow [(\sim p) \wedge r) \rightarrow (p \wedge r)]$.

(ii) Write negation of the following statement: If I am ill, then I cannot go to college.

$3 + 2 = 5$

3. Answer any one question: $10 \times 1 = 10$

(a) (i) Let A, B be two subsets of a universal set. Prove that $A = B$ if and only if $A\Delta B = \phi$.

(ii) For any two sets A and B, prove that $A \cup (A \cap B) = A$.

(iii) A relation ρ is defined on $\mathbb{Z}$ by "$x\rho y$ if and only if $xy \geq 0$" for all $x, y \in \mathbb{Z}$. Examine if ρ is an equivalence relation on $\mathbb{Z}$. 5+2+3

(b) (i) Let p, q, r be statements. Then show that $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$ holds by truth table.

(ii) A relation ρ is defined on the set $\mathbb{Z}$ by "$x\rho y$ if and only if $|x - y| \leq 3$" for $x, y \in \mathbb{Z}$. Examine if ρ is reflexive, symmetric and transitive.

(iii) If ρ is an equivalence relation on a set A, then show that ρ^{-1} is also an equivalence relation on A. 3+3+4

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